ALGORITHMS FOR MODELING THE RANDOM WAVELENGTH OF COMPTON γ-PARTICLE SCATTERING

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The solution of γ-transport problems in matter by the Monte Carlo method involves multiple modeling of Compton scattering. As the energy characteristic of the quantum it is advisable to use the wavelength in Compton units λ, since all the formulas for the Compton process have the simplest and therefore the most economical form from the viewpoint of computer time expenditure.

As is known, the random wavelength λ_n of the n-fold scattered γ-particle is defined on the basis of the Klein–Nishina distribution. To calculate λ_n from given λ_{n-1} and ξ (random number) it is necessary to solve a transcendental equation, which is a very uneconomical operation even with the use of high speed computers. The approximation of the functional relation $\lambda_n = f(\lambda_{n-1}, \xi)$ in bounded intervals of λ variation by certain uncomplicated formulas of the polynomial approximation type in λ_{n-1} and ξ is possible. However, it is clear that use of the exact techniques for calculating λ_n is advisable in order to improve the quality of the statistical simulation.

The exact economical algorithms of modeling λ_n are based on the use of the so-called random "rejection" or exclusion technique. The Kahn algorithm [1] and the algorithm based on the use of the very simple random exclusion technique (see, for example [1]) and realized in $\alpha = \lambda^{-1}$ units in [2] are known.

We shall realize the second algorithm in λ units. To this end we must determine the absolute maximum of the probability density function on the argument variation interval. It can be shown that the maximal value of the Klein–Nishina probability density function

$$k(\lambda_{n-1}, \lambda_n) = \left(\frac{\lambda_{n-1}}{\lambda_n}\right)^2 \left[\frac{\lambda_{n-1}}{\lambda_n} + \frac{\lambda_n}{\lambda_{n-1}} + 2(\lambda_{n-1} - \lambda_n) + (\lambda_{n-1} - \lambda_n)^2\right]$$

on the interval $\lambda_{n-1} \leq \lambda_n \leq \lambda_{n-1} + 2$ equals 2 and is reached for $\lambda_n = \lambda_{n-1}$. In this case the alogrithm for modeling λ_n is:

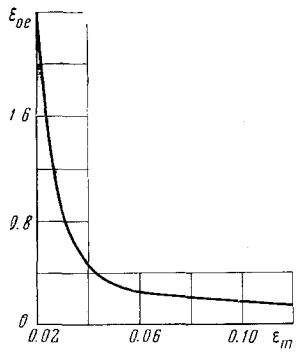

Fig. 1

a) we select two random numbers ξ_1 and ξ_2 and calculate

$$\Lambda_n = \lambda_{n-1} + 2\xi_1;$$

b) we check the inequality

$$2\xi_2 \leqslant k(\lambda_{n-1}, \Lambda_n)$$

If the inequality is satisfied, then $\lambda_n = \Lambda_n$. Otherwise ξ_1 and ξ_2 are rejected and the procedure is repeated with a new pair of random numbers.

It is of practical interest to compare the effectiveness of the Kahn algorithm (denoted I hereafter) and the above algorithm (denoted II hereafter) over a broad energy interval. This was accomplished directly on the computer with determination in the multiple Compton scattering process of the mean time t for degradation of the particle energy from the initial ξ_0 to the final $\varepsilon_{min} = \varepsilon_m$, i.e., we modeled only the "retardation" process and not the particle trajectory as a whole. We followed no less than 1000 such retardations for each set of ε_0 and ε_m.

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Following are the results of determination of the ratio $\tau = t_I/t_{II}$ for the values ε_m = 0.02, 0.05, 0.13 MeV as a function of the values of ε_0, MeV.

$\varepsilon_0 =$	10	8	6	4	3	2.6	2	1	($\varepsilon_m = 0.02$)
$\tau =$	0.79	0.93	0.95	0.96	0.97	1	1.05	1.12	
$\varepsilon_0 =$	0.8	0.6	0.4	0.2	0.1	0.08	0.06	0.04	($\varepsilon_m = 0.02$)
$\tau =$	1.17	1.19	1.22	1.32	1.35	1.37	1.33	1.40	
$\varepsilon_0 =$	10	8	6	4	3	2	1	0.8	($\varepsilon_m = 0.05$)
$\tau =$	0.61	0.62	0.64	0.66	0.72	0.75	0.83	0.90	
$\varepsilon_0 =$	0.6	0.5	0.4	0.32	0.2	0.1	0.08	0.06	($\varepsilon_m = 0.05$)
$\tau =$	0.92	0.95	0.97	1	1.07	1.17	1.30	1.37	
$\varepsilon_0 =$	10	8	6	4	3	2			($\varepsilon_m = 0.13$)
$\tau =$	0.34	0.61	0.45	0.47	0.52	0.57			
$\varepsilon_0 =$	1	0.8	0.6	0.4	0.3	0.2			($\varepsilon_m = 0.13$)
$\tau =$	0.62	0.69	0.75	0.86	0.89	0.98			

The economy of the algorithms I and II depends significantly on ε_0 and ε_m. If $\varepsilon_m \geq 13$ MeV, then for any ε_0 algorithm I is more favorable than algorithm II ($\tau \leq 1$). For each value $\varepsilon_m < 0.13$ MeV there exists a value $\varepsilon_0 = \varepsilon_{0e}$ for which both algorithms are equally economical ($\tau = 1$). With increase of ε_m the value of ε_{0e} shifts in the direction of lower energies. In the case $\varepsilon_0 \leq \varepsilon_{0e}$ it is necessary to use the algorithm II($\tau > 1$).

The data shown in the figure make it possible to find the value of ε_{0e} as a function of $\varepsilon_m < 0.13$ MeV. It is obvious that in the case $\varepsilon_0 > \varepsilon_{0e}$ sequential use of alogrithms I and II respectively is advantageous, particularly for $\varepsilon_0 \gg \varepsilon_{0e}$ and relatively smaller ε_m.

LITERATURE CITED

1. U. Fano, L. Spencer, and M. Berger, Gamma Radiation Transport [Russian translation], Atomozdat Moscow (1963).
2. I. G. Dyad'kin, "Modeling the random energy of a gamma particle scattered as a result of the Compton effect," Zh. vychislit. matem. i matem. fiz., Vol. 6, No. 2 (1966).